Computer Simulation of Ionic Wind in the Point-Torus Electrodes System

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Abstract—It is commonly known that calculation of ionic wind is a difficult task from various points of view. Especially complex task is to take into consideration ionization processes in the sheath of corona discharge. That is why a topic of developing simplified methods of calculation corona discharge and ionic wind is actual. The new simplified method for computation corona discharge is considered. It is based on the unipolar approximation. The negative ions' flux density on the active electrode is defined by the original boundary condition. Comparison of simulation and experimental data (current-voltage characteristic, velocity field) reveals that the simplified model reproduces the basic properties and regularities of ionic wind however quantitative distinctions occur.

Keywords—Ionic wind, corona discharge, computer simulation

I. INTRODUCTION

Ionic wind - air flow caused by corona discharge. This phenomenon is applied in electrostatic precipitators and cooling devices.

Computation of ionic wind requires taking into account aerodynamics of air as well as discharge processes. The described processes have sufficiently different spatial and temporal scales therefore numerical solution is possible [1] but has a large time and memory volume cost. That is why the question of constructing simplified models of ionic wind is relevant.

So called "unipolar" approximation is commonly known and widely used: presence of electrons and positive ions is neglected whereas negative ions are taken into consideration. Corona discharge splits the interelectrode gap into two unequal areas [2]: a small sheath where electrons and positive ions concentration is high and ionization processes are intensive; an external area which occupies the major part of air gap where negative ions concentration is significant but electrons and positive ions concentration may be neglected. Actually using unipolar approximation infers neglecting a finite size of corona discharge sheath.

However there is no charged particles generation in the external area - therefore a boundary condition is introduced instead of the volume source of charged particles (ionization source). The boundary condition describes the negative ions flux which is directed from the sheath to external area. This condition is set on the high voltage electrode surface since there is no sheath in the unipolar approximation.

Different forms of the boundary condition on the negative ions flux are used. Two ways are used for the most part (a representative list of publication on ionic wind simulation is collected in the review [3]).

The first way is to use an experimentally measured current-voltage characteristic. Sometimes the negative ions flux density \( j \) is set as a function of electric field intensity \( E \):

\[
j(E) = C(E - E_0) \quad (1)
\]

The constants \( C \) and \( E_0 \) are fitted so that computed current-voltage characteristic agreed closely with the experimentally measured one. Evidently one of the disadvantages of such way is that the current-voltage characteristic changes with any modification of geometric parameters so one needs to measure it afresh and fit the constants \( C \) and \( E_0 \) again.

The second way consists in using the constant electric field intensity condition on the high voltage electrode surface:

\[
E = \text{const} \quad (2)
\]

The constant is taken close to the critical electric field intensity for air (25 kV/cm) - the Peek's formula [4] is often used for the wire electrode. Evidently the condition (2) should not be set on the whole high voltage electrode (except simple form - e.g. a wire) therefore the corona area should be preliminarily selected on the electrode surface. It constitutes a problem - particularly since the corona area increases with voltage rising.

Also analytical estimations are applied for certain shapes of electrodes - especially for the wire electrode [5].

Completely different conditions are also in occurrence. E.g. space charge on the surface of ionization layer is set as a boundary condition in [6].

Conditions of types (1) and (2) are the simplest forms of relations between negative ions flux \( j \) and electric field intensity \( E \) per se. A relation based on simplified description of processes in the corona discharge sheath is discussed further.

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II. BOUNDARY CONDITION FOR NEGATIVE IONS FLUX

Consider discharge processes in the neighborhood of the high voltage electrode as a series of electron avalanches. Consider the \( i \)-th avalanche. It begins by \( N_i \) electrons emerging from the cathode at the time point \( t_i \). The electrons go through the corona discharge sheath and generate \( P_i \) positive ions and \( P_i \) electrons due to ionization processes where:

\[
P_i = N_i (\exp M - 1)
\]

\[
M = \int_a^b \alpha(E(s)) \, ds
\]  

Here \( \alpha \) - impact ionization coefficient, \( s \) - spatial coordinate along the field line, \( M \) - "multiplication coefficient", \( A \) - the point of intersection of the electric field line and the cathode surface, \( B \) - the point on the electric field line where \( \alpha \approx 0 \) (\( E \approx 25 \text{ kV/cm} \)). Then \( P_i \) positive ions return to the cathode and generate \( \gamma P_i \) electrons due to secondary ion-electron emission process - these electrons are the seed portion for the next \((i+1)\)-th avalanche. Thus:

\[
N_{i+1} = N_i \gamma \left( \exp \left[ \int_a^b \alpha(E) \, ds \right] - 1 \right)
\]  

Here \( \gamma \) - secondary emission coefficient. Electrons generated in the avalanche turn to negative ions due to attachment process. Electrons flux turns to negative ions flux gradually. However here the assumption is made that attachment is intensive and electrons turn to negative ions after leaving the corona discharge sheath instantly. In this case the negative ions number \( Q_i \) which were generated in the \( i \)-th avalanche is:

\[
Q_i = N_i + P_i = N_i \exp M
\]  

Combine (4) and (5):

\[
Q_{i+1} = Q_i \gamma \left( \exp \left[ \int_a^b \alpha(E) \, ds \right] - 1 \right)
\]  

There is a time interval between the \( i \)-th and \((i+1)\)-th avalanches: it is required for the ions to cross the corona discharge sheath. The time interval \( \tau = t_{i+1} - t_i \) is:

\[
\tau = \int_a^b \frac{ds}{\mu E(s)}
\]

So (6) may be rewritten:

\[
Q(t_{i+1}) = Q(t_i) \gamma \left( \exp \left[ \int_a^b \alpha(E) \, ds \right] - 1 \right)
\]  

Suppose that avalanche parameters vary slowly. Then we may represent the left part of (7) as the Taylor series:

\[
Q(t_i + \tau) + \frac{dQ}{dt} = Q(t_i) \gamma \left( \exp \left[ \int_a^b \alpha(E) \, ds \right] - 1 \right)
\]

Finally we get the expression for the changing rate of ions number generated in avalanches:

\[
\frac{dQ}{dt} = Q(t) \gamma \left( \exp \left[ M - 1 \right] - 1 \right) \frac{1}{\tau}
\]

Now suppose that we consider the situation on the small element of the corona sheath surface \( \delta S \). Then negative ions flux density \( j \) amounts:

\[
j = \frac{Q(t)}{\tau \cdot \delta S}
\]

Taking into consideration (4) get the relation for changing rate of \( j \):

\[
\frac{\partial j}{\partial t} = j \left( \frac{e^{u - 1}}{\tau} \right)
\]

\[
M = \int_a^b \alpha(E(s)) \, ds
\]

\[
\tau = \int_a^b \frac{ds}{\mu E(s)}
\]  

Equation (9) may serve as the boundary condition for the negative ions flux on the high voltage electrode surface. The following properties of this condition may be noticed:

1. The commonly known condition of the corona discharge initiation is implied by (9):

\[
\frac{\partial j}{\partial t} > 0 \Rightarrow \gamma (e^{u - 1}) > 1
\]

2. A stationary problem solution does not depend on the parameter \( \tau \) value.

The computer model of ionic wind in the unipolar approximation with (9) as a condition on the high voltage electrode is considered further.

III. SET OF EQUATIONS AND GEOMETRY

The set of equations includes the Poisson's equation for electric potential \( \phi \), transport equation for negative ions concentration \( n \), the Navier–Stokes equations for air velocity \( v \) and pressure \( p \):
\[
\begin{align*}
\Delta \varphi &= \frac{\rho}{\varepsilon_0} \\
\frac{\partial n_+}{\partial t} + \left( \nabla \cdot \left( -D_n \nabla n_+ + \mu_n \nabla \varphi + n_+ \vec{v} \right) \right) &= 0 \\
\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \nabla) \vec{v} &= -\nabla p + \eta \nabla \vec{v} + en \nabla \varphi \\
(\nabla \cdot \vec{v}) &= 0
\end{align*}
\]

(10)

Here: \(D_n\) - diffusivity of negative ions, \(\mu_n\) - mobility of negative ions, \(\rho\) - air density, \(\eta\) - dynamic viscosity of air. Equations (10) are supplemented by boundary conditions:

- **cathode**: \(\frac{\partial \varphi}{\partial t} = j \frac{\gamma(M-1)}{r} \), \(\vec{v} = 0\), \(\varphi = 0\)
- **ground**: \(j = \) open boundary, \(\vec{v} = 0\), \(\varphi = 0\)
- **isolated walls**: \(j = 0\), \(\vec{v} = 0\)

Here \(U(t)\) is voltage.

The point-thorus and sphere-thorus electrodes systems are considered - their schemes are presented in Fig. 1.

**IV. RESULTS AND COMPARISON WITH EXPERIMENT**

The patterns of air flow are presented in Fig. 3. The major jet is directed from the high voltage electrode to the torus. The jet thickness is lower in the point-torus system than in the sphere-torus system (at the certain voltage). The velocity maximum is larger in the point-torus system. Ions go along electric field lines which differ from air flow lines strongly (Fig. 4). Ions distribution in the sphere-torus system is more uniform than in the point-torus system (Fig. 4): in the point-torus system ions are concentrated near the point tip.

The flow rate through the torus opening in dependence on voltage is presented in Fig. 2. It is an integral characteristic of air flow. The flow rate is larger in the point-torus system than in the sphere-torus system by certain voltage.

Compare simulation data with experimental measurement. Current-voltage characteristics are in good agreement in simulation and experiment (Fig. 7).

Velocity field measurements by means of PIV-method which were performed in simulated electrodes systems [7] enable to compare velocity profiles. The experimental and computed profiles are close in form in the sphere-torus system (Fig. 5). However velocity values difference is about 1.5-2 times.

The difference is even more significant in the point-torus system (Fig. 6): both form and velocity values differ. The measured thickness of the jet is 5 times greater than the computed thickness. Velocity values...
difference is about 1.5-2 times. Perhaps these deviations are caused by neglecting the corona discharge size in the used unipolar model.

Experimental data [7] reveal that the sheath size is actually several times larger than the electrode curvature radius in the case of the point-torus system. So the sheath form influences strongly on the flow pattern in this case. On the contrary the sheath size is smaller than the electrode size in the sphere-torus system. Perhaps this is the reason for better agreement in this case.

V. CONCLUSION

The computer model of ionic wind in the unipolar approximation with the original boundary condition for negative ions flux density on high voltage electrode is developed.

Comparison of simulation results with experimental data revealed that a common pattern of the flow is reproduced correctly.

However quantitative distinctions occur. The more is the ratio of the sheath size to the high voltage electrode size the more distinctions are. The computed jet
thickness is close to experimental measurement for the sphere-torus system. However the measured velocity values are 1.5-2 times higher than the computed values. Size and form of corona sheath should be included in the model to reach better agreement.

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