Comparison of Numerical and Experimental Results for the Duct-Type Electrostatic Precipitator

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Abstract—Some fundamental aspects of the electrostatic precipitator are investigated via a combination of tests in a high-voltage rig and numerical computations. The electric field and charge distribution in the duct are computed numerically using the commercially available FEM solver Comsol Multiphysics. The derived properties are compared with experimental data and general operating experience. Overall the agreement between numerical results and experiments is very satisfactory. The numerical solutions provide detailed insight into a number of basic phenomena governing the precipitator behaviour, and may give qualitative guidance also in the practical design of precipitators.

Keywords—Electrostatic precipitator, ESP, numerical, finite-element, FEM, Comsol, experimental

I. INTRODUCTION

The numerical treatment of the electric field and charge distribution within an energized electrostatic precipitator (ESP) has traditionally attracted significant interest. Provided that the boundary conditions at the corona discharge can be simplified the problem formulation is relatively uncomplicated in its most basic form, despite the fact that the coupled partial differential equations (PDE’s) are non-linear. The complexity will increase if more advanced geometries and corona models are considered, or if coupling to the gas flow through the ESP is included in the scope. Also time-dependent problem formulations have been studied, for example to simulate a pulsed power supply. Many aspects of the duct-type precipitator have been investigated numerically in previous works and in addition analytical expressions have been developed within certain approximations. A full review is, however, outside the scope of this paper.

The aim of the present work is to demonstrate the possibility to explain some fundamental phenomena appearing in an ESP via solution of the governing differential equations. This is done numerically using the commercially available program Comsol Multiphysics [1]. This is a user-friendly FEM software package aimed towards coupled systems of partial differential equations. It also has all the necessary features for post-processing, including functionality for visualization of the results in a variety of ways.

Experimental results have been obtained via an ESP pilot in the high-voltage testing facility at the Alstom Environmental Control Systems laboratories in Växjö, Sweden. By this test rig comparisons between numerical solutions and actual data were possible. Although the ESP setup is a "cold pilot" (no hot or dust laden flue gases), care has been taken to reproduce all design detail used in an actual commercial ESP. Furthermore the physical dimension is large enough to avoid excessive boundary effects and allow the power supply to operate at an appropriate current level.

II. NUMERICAL

In a steady state situation there is a constant flow of charge (i.e. ions) from the discharge electrodes towards the collecting plates. Neither the electric field nor the charge density changes with time, implying that the ESP is supplied by an ideal DC supply without ripple or pulsing. For this time-independent approach the coupled PDE’s to be solved are Poisson’s equation (1) for the electric field and the continuity equation (2) for charge conservation:

\[ \nabla \cdot \varepsilon_0 \mathbf{E}(x) = \rho(x) \]  (1)
\[ \nabla \cdot (K \rho(x) \mathbf{E}(x) - D \nabla \rho(x)) = 0 \]  (2)

Here \( \mathbf{E}(x) \) is the electric field and \( \rho(x) \) is the charge density, while \( K \) is the mobility of the charge carriers and \( \varepsilon_0 \) is the permittivity of free space. The position vector \( x \) corresponds to \( (x, y, z) \) in the general 3D case. The diffusion constant \( D \) is in the order of \( 10^{-5} \) m²/s, which makes the diffusion term \(-D \nabla^2 \rho(x)\) in Eq. (2) negligible compared to the convective term [2]. Nevertheless the term may be kept since diffusion typically contributes to numerical stability. In practice the two coupled PDE’s above are often expressed in terms of the electric potential, \( \Phi(x) \), related to the electric field by \( \mathbf{E}(x) = \nabla \Phi(x) \). The corresponding second order equations are also of a typical form for which there exists many numerical schemes. An approximate solution to the above system of PDE’s can be obtained by solving a discretization of the equations either in a fully coupled form or segregated form. In the segregated form the equations are solved separately keeping the other field fixed and iterating between the equations until self-consistency is reached.

The basic geometry of a duct-type precipitator is rather straightforward. In fact, a two-dimensional
numerical treatment is enough to catch most of the physics related to the electric field and charge transport. Specifically for straight wires as discharge electrodes a 2D geometry is an exact mapping of the full three-dimensional problem. Typically the high degree of symmetry within a duct allows for further simplification of the problem via periodic and mirror boundary conditions around the symmetry cell. This is exemplified in Fig. 1, where the most basic 2D geometry is shown together with a solution of Poisson’s equation for \( \rho = 0 \) inside the symmetry cell.

Even if the duct-type ESP in its most basic form is essentially a 2D problem, a full three-dimensional treatment is required for most types of discharge electrodes or if top/bottom boundary effects are studied. An inherent complication is the small dimension of the corona-generating region of any discharge electrode, compared to the spatial extension of the inter-electrode volume. This requires a proper selection of geometry adapted mesh for the FEM discretization.

In order to solve the PDE’s, boundary conditions are needed within the specified geometry. Two of these, for the potential \( \Phi \), are straightforward Dirichlet conditions. The grounded collecting plates gives \( \Phi = 0 \), and the potential at the discharge electrode is taken as the ESP operating voltage \( \Phi = -U \). However, a boundary condition is also needed for the charge density, \( \rho \), and this is a matter of some complexity. The physics of the corona discharge at the emission electrode, where the charge carriers are generated, is so complicated that a detailed treatment is in principle out of the question. Instead some form of phenomenological model must be utilized at the electrode boundary to capture as much as possible of the actual behavior.

A common approach is to use Peek’s semi-empirical condition for corona onset combined with Kaptzov’s assumption. The Peek condition for corona onset postulates that above a critical field strength, \( E_{cr} \), the charge density at the discharge electrode becomes non-zero [3]. Then, as the voltage is raised further, the charge density at the boundary has to increase correspondingly in such a way that the charge supplied to the domain balances the electric field to stay at \( E_{cr} \) (Kaptzov's assumption [4]). Although the Peek-Kaptzov model is straightforward to implement in e.g. cylinder geometry or in a perfectly aligned wire-duct geometry, due to the uniform field around the wire, it is less clear how to proceed for more complicated geometries. In general some sort of injection law must be introduced into the coupled PDE’s, such that the charge density at the discharge electrode surface depends on the local electric field. One of the simplest expressions that may be applied for this purpose is a linear increase of the charge density above the onset field [5]:

\[
\rho(x_r) = \alpha \left( E(x_r) - E_{on} \right) \quad \text{for} \quad E(x_r) > E_{on} \tag{3}
\]

Basically the Peek-Kaptzov condition will follow when the coefficient \( \alpha \) becomes large. Then the simulated corona discharge will act as a copious source of charge and even the smallest increase of the electric field at the boundary will generate the charge needed to quench the field in the self-consistent solution. Due to the strong feedback created by a large \( \alpha \)-value the exact functional relationship of the boundary condition becomes relatively insignificant. Unfortunately the coupled system of PDE’s also becomes increasingly non-linear when \( \alpha \) increases, resulting in convergence problems if the numerical scheme is not set up in a careful way.

The main tool for the numerical computations in this work is Comsol Multiphysics version 4.2 [1]. This is a FEM software package for various physics and engineering applications, especially coupled phenomena. The entered partial differential equations and boundary conditions are collected into one large system that is solved using a weak formulation. Comsol has a selection of direct- and iterative finite element methods for time-dependent equations. Except for the CAD-import and Matlab-link modules, only the base package of Comsol was used in this work, where the equations were entered in the PDE Interfaces menu. All computations in this paper have been performed on a standard PC, equipped with 24 GB RAM and a six-core Intel Xeon 5650 processor at 2.66 GHz.

The results from Comsol have been compared against corresponding solutions from a program developed by Alstom/ABB in the 90’s specifically for numerical ESP simulations [6-8]. Furthermore, three different numerical schemes within Comsol itself were compared for the same benchmark problem, as a further consistency check. One of the Comsol schemes was linking to Matlab code that employed the method of characteristics (MOC) to obtain the charge density [9]. The MOC is the preferred method by many researchers, and was also used in the old Alstom/ABB program [6-8, 10-13]. Fig. 2 shows results from one comparison of the four schemes. The electric potential, \( \Phi \), and charge density, \( \rho \), are plotted along a line that goes straight from the discharge wire directly towards the collecting plate surface. The ion mobility, \( K \), is assumed to be \( 1.8 \times 10^{-4} \text{ m}^2/\text{Vs} \), in accordance with experimental findings [14]. By choosing the collecting plate spacing, wire pitch and wire radius in the ideal geometry to \( 9'' \), \( 6'' \) and 0.044'', respectively, the numerical solution for the electric potential in Fig. 2 could also be compared.

Fig. 1. Basic 2D geometry for duct-type ESP. The domain treated numerically is shown with a solution for the electric potential. The size of the discharge wires has been increased for the sake of clarity.
with experimental results by Penney and Matick [15]. The applied voltage is 46.2 kV.

It is seen that all numerical schemes generate identical results (indistinguishable curves), and that the potential is in excellent agreement with the measured data. It is also worthwhile to mention that the Penney-Matick experiments gave an average current density of $688 \mu A/m^2$ for the voltage at hand, while the computations result in $725 \mu A/m^2$. The relatively good agreement indicates that also the charge density from the computations may be representative of the real situation. For the computations seen in Fig. 2 the value of the critical field strength, $E_{on}$, has been taken as $6.1 \times 10^6$ V/m, which follows directly from Peeks formula [3] for ambient temperature and a wire radius of 0.04”.

The similarity between the different numerical schemes, including MOC variants, indicated by Fig. 2 is remarkable. The fully coupled solver in Comsol turned out to be robust and convenient to use, and is therefore the basis for the numerical computations in this work.

### III. EXPERIMENTAL SETUP

To be able to compare the numerical results against actual experimental data a duct-type ESP has been constructed. It consists of two parallel gas passes with variable spacing and can be fitted with either helical wires (spirals) or straight wires as discharge electrodes. Each collecting electrode curtain consists of four profiled plates with a nominal width of 800 mm (i.e. resulting in a field length of 3.2 m). The height of the collecting plates is 4 m, leading to a nominal collecting area of 51.2 m². The wire/spiral diameter is 2.7 mm, giving $E_{on} = 5.7 \times 10^6$ V/m according to Peeks formula [3].

Since one of the purposes of the ESP test rig is to investigate critical areas for spark-over, it is constructed as a replica of a standard Alstom commercial precipitator. Thus, even if it is a cold pilot, it includes details like support beams, shock bars, tumbling hammers and rapping shafts. Fig. 3 shows a photo of the rig. The ESP can be energized either by a conventional transformer-rectifier (T/R) or by a high frequency power converter (SIR). The conventional single-phase T/R is rated 200 mA and 200 kV. The SIR (Switched Integrated Rectifier) converts the three-phase AC at the mains to a rectified high frequency, high-voltage output [16]. This leads to nearly perfect DC voltage, compared to the rippled DC from a conventional T/R. The rating of the particular SIR used for the ESP rig is 200 mA, 125 kVp.
The voltage on the discharge frame was measured using an external wideband HV voltage divider with very fast time response (Ross Engineering VD180-8.3Y-AK). With this equipment voltage ripple, sparking transients, etc. could be accurately studied with oscilloscope. Concerning the current measurement this was, in addition to the internal readings of the SIR or T/R controllers, obtained by a voltage reading across a calibrated current shunt between collecting plates and ground. A feature later added to HV-rig was the possibility to measure the current distribution profile on the collecting plate via a special foil glued to one collecting plate. The foil consisted of one row of 1 cm² squares, each connected to one logger with 100 channels. By raising the entire collecting plates in steps of 1 cm the current profile on the plate could be mapped. The data generated by this equipment is compared against numerical results in Sec. IV-C. Similar current distribution measurements have also been performed previously in the Alstom high-voltage lab [6, 17].

IV. RESULTS & DISCUSSION

A selection of results from the numerical Comsol computations of relevance for a practical precipitator are described in the below sections. Where possible the numerical results have been compared to experimental data or general field experience. Although voltages and charges are given as absolute numbers, negative corona is implied throughout.

A. Profiled Collecting Plates

To reach the mechanical stability required for a tall collecting plate it must be profiled. Since the electrical field strength at the grounded plate surface is postulated to determine when a spark-over occurs, it is very important to have smooth curvatures [18].

Comsol has functionality to import CAD drawings, which was convenient for building the geometry for the plates. Compared to the ideal geometry the number of mesh points increases significantly at the plate to be able to resolve the electrical field at the “G-profiles” at the plate ends. Furthermore, the symmetry cell now includes all three wires and both collecting plates. This can be appreciated from Fig. 4, where the electric field strength is shown as a color map. It is seen that the electric field is significantly enhanced at the perturbations on the collecting plates, which is in accordance with previous findings [18].

When comparing the numerically computed maximum field strength at the G-profiles for conditions matching the spark-over limit in the experimental rig, it was found that it was virtually independent of geometry (e.g. plate spacing). Thus, the combined results from numerical computations and experimental spark-over voltages support the assumption that it is indeed the maximum local electrical field strength at the grounded plates that determines the spark limit in a precipitator. This critical field strength, occurring at the upper G-profiles to the left and right in Fig. 4, was approximately $1.3 \times 10^6$ V/m. It should be pointed out, however, that experimentally it is rather difficult to determine a unique point where consistent spark limited operation has developed. Another complication is that the 125 kV capacity of the SIR-unit was only enough to reach sparking at 300 mm plate spacing. For wider spacings only the conventional T/R produced sparks, and the significant voltage ripple may not be adequately covered within the numerical steady state model.

B. Current-Voltage Characteristics

An important diagnostic of a precipitator is the current-voltage characteristics, or I-V curve. The reproduction of experimental I-V curves within the numerical treatment of an ESP is thus an important consistency check of any model.

Fig. 5 compares experimental I-V curves generated by both the conventional T/R and the SIR power supply with computed curves from Comsol. Also the difference in I-V characteristics between spirals and straight wires obtained in the test rig is shown (for the SIR power supply). The experimental data shows that for a given average current input the average voltage is somewhat

Fig. 4. Electric field strength of a solution with profiled 800 mm collecting plates. For the rightmost wire field lines are also included.
higher for the SIR compared to the conventional T/R. However, the peak voltage for a given current is of course significantly higher for the conventional T/R due to the ripple. For example, at an average voltage of 70 kV$_{av}$, the peak voltage was measured to be 110 kV$_p$ for the T/R. For the SIR the corresponding value was about 75 kV$_p$. From this it is easily understood that a significantly higher power input can be reached before spark-over. It should be noted that the high ripple of the T/R unit was measured with the external high-voltage probe with very high bandwidth. The voltage readings of the T/R controller, based on a current shunt with a conventional resistor, indicated much lower ripple. This is due to the rather slow time response of the shunt, which is only designed to measure an accurate average voltage. Another finding from the HV-rig is that the voltage is about 5 kV higher for straight wires compared to spirals for any given current.

The numerical results, shown as dashed lines in Fig. 5, demonstrate that the best agreement between the computations and experiments is obtained for the case of straight wires and SIR. This is expected, since the numerical model employed does indeed correspond to straight wires (2D model) and perfect DC current (stationary model). The significant time dependence due to the 50 Hz ripple from a T/R cannot be properly represented in our stationary numerical treatment. Therefore it is seen that even when the corona onset from Peek’s experiments is artificially lowered by as much as 20% the curve shape does not satisfactorily fit the data obtained from the T/R. It is worthwhile to emphasize that the good agreement between SIR data with wires and the numerical 2D results was obtained for values of $E_{on}$ and $K$ that have previously been verified experimentally [14].

C. Current Distribution on the Collecting Plate

A good current distribution (low coefficient of variation) on the collecting plates is important in a real precipitator e.g. to minimize back-corona in the case of high resistivity dust [17]. With the measurement foil glued to one of the collecting plates in the HV-rig the current entering each cm$^2$ of the collecting area could be measured. Corresponding information from the numerical solutions can easily be obtained from the post processing tools in Comsol. Using straight wires (and SIR power supply) for the experiment, the 2D treatment in Comsol suffices for an accurate representation of the actual geometry.

Fig. 6 shows a comparison between an experimental measurement and a numerical computation at an average current density of 500 $\mu$A/m$^2$. The agreement between experiment and computation is seen to be quite good. The three "humps", showing the higher current density in front of each wire are well reproduced, as is the sharp peaks at the G-profiles at the collecting plate ends. The high current at the plate perturbations is a direct reflection of the high electrical field strength at these positions (c.f. Fig. 4).

D. Estimate of Migration Velocity

The classical measure of ESP performance is the Deutsch migration velocity, $\omega$, which may be seen as the speed at which the particles travel towards the collecting plates [19]. The ESP collection efficiency is then given by

$$1 - C_{out}/C_{in} = 1 - \exp\left[-\omega A/Q\right]$$

(4)
where \( A/Q \) is the specific collecting area in \( m^2/(m^3/s) \). The equilibrium velocity of a charged particle of radius \( a \) in the electric field is given by the balance of the driving force and the drag force:

\[
\omega \propto \frac{qE_y}{\pi \eta a}
\]

The saturation charge on a particle can easily be derived in the field-charging regime [19]. The result in SI units for a perfectly conducting spherical particle is

\[
q = 12 \pi \varepsilon_0 a^2 \| E \|
\]

From the numerical solution the electric field is known at each point in the inter-electrode space. Assuming that the turbulence and gas flow gives no net contribution to the transport of a particle towards the collecting plates, the average velocity is given by:

\[
\omega = \frac{2 \varepsilon_0 a}{\eta} \int_{\Omega} \| E \| dE_y(x) \left/ \int_{\Omega} dx \right.
\]

The charging field, \( |E| \), is taken as:

\[
|E| = \int_{\Omega} dx |E(x)| \left/ \int_{\Omega} dx \right.
\]

The integrals in Eqs. (7) and (8) can be performed by the post processing tools in Comsol on the solution domain, \( \Omega \), for the geometry and boundary condition (voltage) at hand.

Since there has been some debate in the past regarding the influence of plate spacing on migration velocity the evaluation has been done for a number of geometries, having different spacing. The computations have been repeated for several current densities for each spacing, assuming particles of diameter 2 \( \mu \)m.\( (a = 1 \times 10^{-6}) \) and a gas viscosity, \( \eta \), of 1.8 \times 10^{-5}. The result is shown in Fig. 7.

As expected the migration velocity increases with increasing current density (and associated voltage). The curves represent 50, 150, 300, 500, 1000, 1250 and 2000 \( \mu \)A/m², respectively. It is also seen that the migration velocity increases strongly with the plate spacing divided by the migration velocity.

Fig. 6. Experimental and numerical current profile entering the collecting plate. The average current input is 500 \( \mu \)A/m².

Fig. 7. Migration velocity as function of plate spacing at different current densities. The “migration time” in the right graph is defined as half the plate spacing divided by the migration velocity.
spacing for a given current density. The dashed curve in Fig. 7, having a current density of 1250 μA/m², represents the break line where the migration velocity becomes proportional to the plate spacing. Proportionality would nominally correspond to a particle collection efficiency that is independent of spacing. This is illustrated in the right graph of Fig. 7, showing the "migration time" (here taken simply as half the spacing divided by the migration velocity). Already at e.g. 500 μA/m² the migration time has a very weak dependence on the spacing. The almost linear increase of migration velocity with spacing for constant current density is in accordance with semi-analytical/experimental results by Castle et al. [20]. A point of notice when considering trends of constant current is the tendency for the precipitator to reach the spark-over limit at a lower current density when the plate spacing is increased. This is due to the relative enhancement of the electric field at the plates and was seen experimentally in the ESP rig and correlates with the numerical findings of Sec. IV-A above.

It is important to emphasize that the above measure for the migration velocity is only a rough qualitative estimate. From the full numerical solution a number of other variants could be envisioned, such as for example limiting the integration to an area adjacent to the collecting plate where the actual collection of particles takes place. A more refined model is to add also the gas flow inside the ESP, including turbulence model and volume force from the ion transport, and thereafter run particle trajectories through the domain. The fraction of particles that reach the far end of the domain compared to those impacting on the collecting plates gives an estimate of the collection efficiency [8]. However, even the most advanced model would likely not be able to provide a practical estimate of a that may be used e.g. for ESP sizing.

E. Impact of Space Charge due to Dust Laden Gas

The phenomenon of corona suppression due to the space charge effect from a large amount of submicron particles in the gas is a well-known problem for certain ESP applications, such as soda recovery boilers [21]. Also for other applications, as for example precipitation of fly ash after a coal-fired boiler, it is typically seen that the first field of the ESP has to operate at a higher voltage to reach a given set point for the current. Often this also leads to excess sparking in the front field.

The simplest possible model to take into account the dust space charge is to add a constant contribution to the charge density in the entire domain [18-19, 21]. This contribution does not add to the current due to the very low migration velocity of the particles compared to the charge carried by the fast-moving ions. In this simplest approach Poisson’s equation thus takes the form

$$\nabla \cdot \varepsilon_0 E(x) = \rho(x) + \rho_{\text{dust}} \quad (9)$$

while the continuity equation, Eq. (2), for $\rho(x)$ remains unchanged (though $\rho_{\text{dust}}$ enters implicitly by affecting $E$).

When this exercise is done in the numerical 2D model in Comsol it is clearly seen how, for a given voltage, the current decreases when $\rho_{\text{dust}}$ becomes larger. This is demonstrated in Fig. 8 for the voltages 60 kV and 80 kV at a plate spacing of 400 mm.

It is interesting to note that complete corona quenching occurs already at a dust space charge of 12 μC/m³ and 21 μC/m³ for 60 kV and 80 kV, respectively. This is considerably lower than the saturation charge for typical dust concentrations at the ESP inlet. For example, the virgin fly ash from a pulverized coal boiler may have a surface area in the order of 1 m²/g. For a typical inlet dust concentration to the ESP of 10 g/m³ this would result in a saturation charge of >100 μC/m³ for a reasonable average charging field (c.f. Eq. (6)). Thus the particles cannot obtain maximum charge before a significant fraction of them has disappeared from the inter-electrode region by precipitation. Only then can the remaining particles reach their full saturation charge without completely quenching the corona.

Another model, which is somewhat more advanced than a constant dust charge, is to take an arbitrary distribution along the precipitator length. Thus any explicit function $\rho_{\text{dust}}(x)$ may be defined. If a reasonable shape is selected this can for example simulate the decrease of $\rho_{\text{dust}}$ due to collection of particles. This was performed for a numerical 2D model with parameters loosely based on recent data from an ESP after a lignite-fired oxyfuel boiler [22]. The function was taken to be of the form $\tilde{\rho}(x) \exp[-Ax]$, where $\rho(x)$ is a smoothed step function to simulate the charging of the particles, and $A$ is selected to represent the decrease of the dust (charge) concentration through the first field of the ESP. The maximum dust charge, $\tilde{\rho}$, was selected as the charge density for complete corona quench, which is the 12 μC/m³ from Fig. 8 since the operating voltage was around 60 kV for the present example. From dust measurements at the oxyfuel plant it was concluded that the dust concentration after the first ESP field was in the order of 20 mg/m³ at the high current density prevailing (450 μA/m²). For such a low dust concentration the particles can have their saturation charge without
significant corona quenching, even though their average diameter is very small. A dust space charge of maximum a few \( \mu \text{C/m}^3 \) may be expected. Based on this an appropriate value for \( \lambda \) can be chosen.

The so-obtained function for \( \rho_{\text{dust}}(x) \) was added to a 2D model stretching over the full length of the first ESP field. Since the discharge wires will have a current output that depends on their position relative \( \rho_{\text{dust}}(x) \) the Peek-Kaptzov condition could not be enforced explicitly. Instead the charge injection law according to Eq. (3) was used with a large \( \alpha \)-value to reach a pseudo Peek-Kaptzov assumption (see Sec. IV-F for further details).

The numerical result showed that the voltage needed to reach the set point of 450 \( \mu \text{A/m}^2 \) was about 8 kV higher compared to the case when no dust space charge was added. This correlates reasonably well with the voltage difference of about 5 kV between the first and second ESP field that was observed during the measurement campaign at the oxyfuel plant [22].

Of course the selected ad-hoc expression for \( \rho_{\text{dust}}(x) \) above is rather arbitrary, but may serve as a first approximation to indicate how a dust space charge that varies along the ESP can affect the operating voltage. It also demonstrates how the use of an injection law in the numerical scheme will lead to a solution that is adapted to the varying dust space charge, as shown in Fig. 9. A first step towards refinement of the model would be more realistic approximations for the charging rate and the precipitation rate. For example, the charging of particles is treated in a quite detailed way in recent numerical work by Adamiak and Atten [23]. A desirable approach would be to develop a transport equation for \( \rho_{\text{dust}}(x) \), included in the system of PDE’s. In this way it could be appropriately coupled to the ionic charge and electric field to reach a self-consistent solution. However, since \( \rho_{\text{dust}}(x) \) is inherently dependent on the dust concentration, assumptions regarding the particle collection would still have to be made in lieu of a full inclusion of also the dust concentration, \( C(x) \), within the PDE’s.

### F. 3D-Computations for Spiral Discharge Electrodes

Although it has been shown by the previous sections that much of the physics of a duct-type ESP can be adequately covered using 2D models, some computations have also been carried out in 3D. This is exemplified below for spiral discharge electrodes, which are often used in commercial full-scale ESPs due to the favorable current distribution compared to e.g. peak electrodes. A 3D-treatment requires significantly higher number of mesh points and degrees of freedom in the numerical model, making memory and computation time an important factor to consider. With the 24 GB memory available it was possible to handle about 320000 elements of second order with a fully coupled solver (resulting in roughly one million degrees of freedom).

The geometry covered one turn of one spiral with non-profiled plates, using an unstructured boundary fitted tetrahedral mesh. About 80% of the mesh points were located in the immediate vicinity of the spiral electrode and for future refined studies it may be beneficial to consider a structured mesh, or boundary layer, closest to the spiral.

Due to the highly non-uniform electric field at the spiral surface an injection law must be used, e.g. as per Eq. (3), described in Sec. II. Given the successful implementation of the Peek-Kaptzov condition for the 2D computations, it was desired to use a similar approach also in the 3D case. This means using a large value of the \( \alpha \)-parameter in Eq. (3). The definition of "large" is related to how much the electric field in the active corona region on the electrode surface is allowed to deviate from the Peek value, \( E_{\text{on}} \). Combining the current flow out from a straight wire, given by \( i = 2\pi r_0 K \| E(x_0) \| \rho(x_0) \), with the injection law in Eq. (3) it can be seen how the relative variation of the field on the wire surface depends on \( \alpha \):

\[
\frac{\| E(x) \| - E_{\text{on}}}{\| E(x_0) \|} \approx \frac{i}{\alpha 2\pi r_0 KE_{\text{on}}} \quad (10)
\]

Thus \( \alpha \gg i_{\text{max}}/2\pi r_0 K E_{\text{on}}^2 \) is the criteria for a pseudo Peek-Kaptzov treatment, where \( i_{\text{max}} \) is the highest current to be handled (in A/m of discharge wire) and \( r_0 \) is the wire radius. For discharge electrodes other than wires the circumference \( 2\pi r_0 \) should be exchanged by some effective coronating area per meter of electrode. A convergence study for \( \alpha \) is presented in Table I for wire

<table>
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<th>( \alpha [\text{C/Vm}^2] )</th>
<th>2D ideal</th>
<th>2D G-plate</th>
<th>3D spiral</th>
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<td>149 ( \mu \text{A/m}^2 )</td>
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<td>( 5.31\times10^{-10} )</td>
<td>159 ( \mu \text{A/m}^2 )</td>
<td>186 ( \mu \text{A/m}^2 )</td>
<td>164 ( \mu \text{A/m}^2 )</td>
</tr>
<tr>
<td>( 5.72 \text{ MV/m} )</td>
<td>5.72 ( \text{ MV/m} )</td>
<td>5.77 ( \text{ MV/m} )</td>
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and spiral electrodes. The average electric field in the corona region is shown together with the resulting current density. The largest $\alpha$-value in Table I of $5.31 \times 10^{-10}$ is about 300 times larger than $i/2\pi r_0 K E_{on}^2$, with $E_{on}$ for the 2.7 mm wire/spiral being 5.7 MV/m.

The boundary condition of Eq. (3) could be directly implemented in Comsol as a constraint for the 2D computations, but for the 3D computations implicit use of the constraint led to "Out of memory" problems. Instead the formula in Eq. (3) was treated as an explicit Dirichlet condition for the charge density on the spiral surface. The PDE system, Eq. (1) and Eq. (2), was then solved repeatedly with a fully coupled solver, using manual restarts with under-relaxation until self-consistency was reached. The geometry and numerical solution for the 3D spiral can be appreciated from Fig. 10.

The voltage on the spiral is 70 kV and the $\alpha$-value is $5.31 \times 10^{-10}$ C/Vm$^2$. From the cut-plane $z = 0$, displaying the charge density, it is seen how the corona discharge is injecting current to the domain from the "outer" side of the spiral surface, where the electric field is strongest (above $E_{on}$). That the corona discharge is located on the outer side of the spiral is clearly seen in the experimental test rig. This is shown by the photograph in Fig. 11, taken in darkness with long exposure time. Back to the numerical solution in Fig. 10, it demonstrates the nodal pattern of the current distribution entering the collecting plates, which were confirmed experimentally in the HV-rig as well as in preceding investigations [6, 17]. In field inspections of full-scale ESPs with spiral electrodes evidence of the nodal pattern is sometimes also seen as thicker dust deposits on the collecting plates opposing the bulges of the spiral.

Regarding the current-voltage characteristics the computations did not reproduce the lower voltage for a spiral compared to a straight wire to reach a given current. The voltage difference was about 5 kV as per the experimental $I$-$V$ curves in Sec. IV-B, while only marginal difference between spiral and wire could be seen in the numerical computations for ideal geometry (c.f. Table I). On the other hand, solving Laplace equation (i.e. Eq. (1) with $\rho = 0$) for spiral and wire, respectively, results in a corona onset voltage that is about 6 kV lower for the spiral compared to a straight wire. It is actually somewhat peculiar that this difference does no longer manifest itself when corona current is included in the numerical model via the coupled system of Eq. (1), Eq. (2) and Eq. (3). This should be investigated further, and may lead to the need of a refined treatment of the injection law.

V. CONCLUSION

By a number of examples it has been shown that relatively simple numerical models can be used for an increased understanding of various phenomena in the duct-type ESP. For this purpose it is very often enough with a 2D treatment. Satisfactory agreement with experiments and field experience was reached. For the type of investigations performed in the present work it is possible to use commercially available FEM solvers, like Comsol Multiphysics, and regarding the hardware it is nowadays enough with a state-of-the-art PC.

Some highlights on the experimental results are the measured 5 kV difference in $I$-$V$ characteristics between spiral and straight wire electrodes, as well as the recorded current distribution profile on the collecting plate for straight wires. Also the very significant voltage ripple from the conventional T/R set measured with a wide-band high voltage probe is worth to point out. On the numerical side the accurate replication of $I$-$V$ curves and current distribution by 2D models for experimental results with wires and non-rippled voltage supply is encouraging. Furthermore application of a relatively straightforward charge injection law was shown to produce a Peek-Kaptzov behaviour when the coupling coefficient, $\alpha$, in the boundary condition on the discharge electrode was sufficiently large. Finally, a simple estimate of the migration velocity $\omega$ indicated a strong increase with plate-to-plate spacing, provided that current density is constant.
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REFERENCES