Corona Current-Voltage Characteristics in Wire-Duct Electrostatic Precipitators “Theory versus Experiment”

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Abstract—This paper is aimed at investigating how the corona current-voltage characteristics in an electrostatic precipitator (ESP) is influenced by the number of discharge wires, the wires’ radius and the spacing between wires and the collecting plates. The corona current-voltage characteristics are calculated for each wire. The corona current-voltage characteristics are measured for a laboratory model of a precipitator and calculated by simultaneous solution of Poisson’s equation, current density equation and continuity equation for current density. The calculated corona current-voltage characteristics values agreed reasonably with those measured experimentally.

Keywords—Corona, current-voltage characteristics, wire-duct, electrostatic precipitator

I. INTRODUCTION

Electrostatic precipitators (ESP) are employed in electric power plants and many industries such as cement production, chemical processing, and domestic air cleaning. The basic processes of electrostatic precipitation are straightforward and well known [1]. A pure theoretical method was presented [2] for calculating the corona current in conductor-to-plane gaps representing unipolar dc transmission line configurations. This method is applied in the present work to investigate corona current-voltage characteristics in wire-duct electrostatic precipitators. An accurate knowledge of the electrostatic field distribution is requested for determining the corona current from discharge wires in precipitators [3, 4]. The onset voltage of corona is a prerequisite for calculating the corona current at a given voltage applied to the discharge wires. The onset voltage of corona from discharge wires in wire-duct precipitators was investigated before [5, 6].

This paper is aimed at investigating how the corona current-voltage characteristic in a wire-duct electrostatic precipitator is influenced by the number \(N\) of discharge wires, the wires’ radius \(r_c\) and the spacing between the collecting plates \(2H\), Fig. 1. The corona current-voltage characteristics are calculated for each wire. The corona current-voltage characteristics are measured for a laboratory model of a precipitator and calculated by simultaneous solution of Poisson’s equation, current density equation and continuity equation for current density. The calculated corona current-voltage characteristics values agreed reasonably with those measured experimentally.

II. GOVERNING EQUATIONS

Poisson’s equation relates the electric field intensity \(E\) to the space charge density \(\rho\) as:

\[
\nabla \cdot E = \frac{\rho}{\varepsilon_0} \tag{1}
\]

where \(\varepsilon_0\) is the permittivity of free space. At the same time, the continuity for the current density or ionic flow vector \(J\) is homogeneous:

\[
\nabla \cdot J = 0 \tag{2}
\]

The ionic flow vector is determined by the electric field as:

\[
J = k \rho E \tag{3}
\]

where \(k\) is the mobility of the ions. Defining the scalar potential \(\phi\), which is related to the electric field intensity by:

\[
E = -\nabla \phi \tag{4}
\]

All attempts in the literature were based on some simplifying assumptions [2, 4]. The most common ones are as follows:

(a) The electrode spacing is filled with unipolar space charge like wires’ polarity.
(b) The space charge affects only the magnitude not the direction of the electric field, i.e.,

\[
E = \lambda \xi \tag{5}
\]

where \(\lambda\) is a scalar point function of space coordinates depending on charge distribution and \(\xi\) is the space-charge-free field. The calculation of the space-charge-free-field within the inter-electrode spacing between the discharge wires and the collecting plates was reported before [6].
(c) The mobility of ions is constant.
(d) Thermal diffusion is neglected.
(e) The surface field of the discharge wires remains constant at the onset value \(E_o\), which was calculated before [6].
III. METHOD OF ANALYSIS

A. Electric field and space charge distributions

A method is developed based on mathematical manipulation of equations (1) through (5) which ends up with the following equations for the distributions of the charge density $\rho$ and the scalar $\lambda$ along the streamlines of the electric field between the discharge wires and the collecting plates:

$$\phi = \int \frac{1}{\rho} \left[ 1 - \frac{2}{e_0 \rho_0} \frac{E}{\rho^{1/2}} \right] \phi \, d\phi$$  

$$\lambda^2 = \lambda_0^2 + 2 \rho_0 \lambda_0 \int \frac{E}{\rho^{1/2}} \, d\phi$$

where $\rho_0$ and $\lambda_0 = V_0/V$ are the values of $\rho$ and $\lambda$ at the surface of discharge wires. $V$ and $V_0$ are the applied and corona-onset voltages.

B. Corona current calculation

For each of value of the applied voltage above the onset value $V_0$, the corona current is equal to the sum of the currents flowing in all flux-tubes extending between the discharge wires and the collecting plates. The corona current density $J_{in}$ at the surface of the discharge wire where the nth flux-line extending along the axis of the nth flux tube is obtained using eqn. (3), i.e.,

$$J_{in} = k \rho_{in} E_{in}$$  

where $\rho_{in}$ and $E_{in}$ correspond to the values of $\rho_e$ and $E_0$ for the nth flux line.

The corona current density $J$ averaged over the wire periphery is calculated as:

$$J = \left( \sum_{n=0}^{N} J_{in} \right) / M$$

where $M$ represents the flux tubes emanating from the discharge wire. Multiplying the average value of current density obtained by eqn. (8) by the surface area per unit length of the wire yields the value of the wire corona current. Summation of the corona current per wire for all ESP wires determines the corona current $I$ at the applied voltage value $V$.

IV. FLOW CHART OF THE COMPUTER PROGRAM

Fig. 2 shows a flow chart describing the procedure based on MATLAB computer program for calculating the corona current per discharge wire. The corona currents are calculated for applied voltages starting at $V_s$ ($> V_0$) and increased in steps $\Delta V$ up to a maximum voltage $V_M$. Because of symmetry around both x- and y-axes, it is sufficient if the current distribution is calculated only for one half of ESP, Fig. 1.

V. CALCULATED CORONA CURRENT-VOLTAGE CHARACTERISTICS AGAINST PREVIOUS CALCULATIONS

The current-voltage characteristic for one-wire ESP (wire radius = 1 mm and wire-to-plate spacing = 11.43 cm) is shown in Fig. 3. It is clear from this figure that the proposed method predicts current values very close to those obtained before using boundary elements [3] and finite elements [4] for the same applied voltage. The proposed method is also applied for calculating the current-voltage characteristics of a three-wire ESP (wire radius = 0.521 mm, wire-to-wire separation = 20.32 cm, and wire-to-plate spacing = 10.16 cm) is shown in Fig. 4. It is quite satisfactory that the calculated corona current-voltage characteristics agreed well with those obtained before [4, 7].

VI. EXPERIMENTAL CHECK-UP OF CALCULATED CORONA CURRENT-VOLTAGE CHARACTERISTICS

A. Experimental Set-up

A wire-duct electrostatic precipitator (ESP) was set-up in the HV laboratory of Czech Technical University in
A schematic diagram of the experimental set-up is shown in Fig. 5. The set-up consists of: A regulating transformer, whose input is 220 V, feeds the HV circuit through a contactor switch to connect or disconnect the supply.

A HV transformer to step-up the output voltage of the regulating transformer. The output voltage of the HV transformer is rectified through a rectifier circuit being immersed in the transformer oil with a smoothing capacitor bank consisting of two series capacitors; each one is 0.25 µF, 100 kV. The generated DC voltage is variable in the range 0-200 kV and is applied to the investigated wire-duct ESP through an 80 kΩ resistance to limit the current in case a flash happens in ESP.

Discharge wires of the ESP itself are steel conductors with radius $r_c$ of 0.26, 0.935 and 1.975 mm. The wires are hanged vertically between two collecting plates. Each wire is terminated at both ends by two smoothing steel spheres to avoid field intensification. The top ends of the wires are stressed through smooth steel strip. The wire-to-wire spacing $d$ is adjusted to 14.5 cm.

Two collecting plates forming the duct of the ESP are made of steel and hanged vertically from a steel support. The dimensions of each plate are 125 cm height and 250 cm length. The plate to plate spacing ($2H$) is adjusted to 30 cm and 40 cm. All edges of the collecting plates are curved outside to avoid field concentration at edges.

A micro-ammeter is used for measuring the corona current from the discharge wire. The meter is shielded to avoid local corona effects. The reading of the micro-
ammeter is recorded by a digital camera which is connected to a computer to record the reading.

B. Experimental technique

To measure the corona current-voltage characteristics from each discharge wire, a shielded micro-ammeter is connected to the discharge wire and the corona current is recorded as described above with the increase of the applied voltage.

C. Experimental check-up results and discussions

The corona current-voltage characteristics were measured in the laboratory for each discharge wire of ESP. The corona currents of each discharge wires were measured for different voltage values applied to the discharge wires. Different values of wire radius (rc = 0.52, 1.87 and 3.95 mm) were used for one- and multi-discharge wires (3, 5 and 7 wires) with plate-to-plate spacings (2H) adjusted for 30 and 40 cm.

All the measurements are made in HV laboratory of pressure = 1001.3 kPa and temperature = 22°C.

The maximum voltage applied to the set-up circuit without flash-over between HV feeding terminals and steel supporting frame is about 120 kV. Therefore, the applied voltage can be increased safely up to 110 kV.

D. Error currents

Fig. 6 shows the currents flowing through the set-up when which the discharge wires were disconnected. These currents are in the micro-ampere range which is very small when compared with the milli-ampere range of the ESP corona currents in the presence of discharge wires. This confirms that the set-up connection is free from corona and corona takes place only at the discharge wires of the precipitator.

E. Corona current-voltage characteristics

Corona current-voltage characteristics were measured and calculated for wire-duct precipitators with different discharge wires (1, 3, 5 and 7) at varying wire radius and plate-to-plate spacing as shown in Figs. 8 through 28. The wire-to-wire spacing was kept constant at 14.5 cm. It is satisfying that the calculated characteristics agreed reasonably -within the experimental scatter- with those measured experimentally.

E.1. Effect of wire radius (rc)

The smaller the wire radius, the smaller the onset voltage and the higher is the corona current at the same applied voltage and plate-to-plate spacing for a single-wire precipitator, Figs. 8-10. Also, the smaller the wire radius, the smaller the onset voltage and the higher is the central-wire corona current at the same applied voltage and plate-to-plate spacing for 3-wire precipitators, Figs. 11-13, 5-wire precipitators, Figs. 14-16 and 7-wire precipitators, Figs. 17-19. The decrease of the corona onset voltage with the decrease of the wire radius is attributed to the corresponding field enhancement at the wire surface [5, 6]. The corona current depends on how high the applied voltage above the onset value. This is why the corona current at the same applied voltage increases with the decrease of the onset voltage.

E.2. Effect of plate-to-plate spacing (2H)

The larger the plate-to-plate spacing, the higher is the onset voltage and the smaller is the corona current at the same applied voltage and wire radius for a single-wire precipitator, Figs. 8-10. Also, the larger the plate-to-plate spacing, the higher is the central-wire corona current at the same applied voltage and wire radius for 3-wire precipitators, Figs. 11-13, 5-wire precipitators, Figs. 14-16 and 7-wire precipitators, Figs. 17-19. It is quite clear that the corona current decreases with the increase of plate-to-plate spacing, Figs. 8 - 19, for the same applied voltage and wire radius. This is simply explained by the resulting
decrease of the electric field along the flux lines, where the corona ions are convicting between the discharge wires and the collecting plates. The more of plate-to-plate spacing the more is the decrease of the electric field along the flux lines with subsequent decrease of the corona current for the same applied voltage in agreement with Fig. 8-19.

E.3. Effect of number of discharge wires (N)

The larger the number of discharge wires, the more is the shielding effect imposed on the central wire with a subsequent increase of the corona onset voltage and decrease of corona current emitted from the central wire at the same applied voltage and plate-to-plate spacing. Figs. 8, 11, 14 and 17 for \( N = 1, 3, 5 \) and \( 7 \) at \( r_c = 0.26 \) mm, Figs. 9, 12, 15 and 18 for \( N = 1, 3, 5 \) and \( 7 \) at \( r_c = 0.935 \) mm and Figs. 10, 13, 16 and 19 for \( N = 1, 3, 5 \) and 7 at \( r_c = 1.975 \) mm.

Figs. 20 - 22 show the corona current-voltage characteristics of central and outer wires of 3-wire precipitator as influenced by the wire radius at the same plate-to-plate spacing. Figs. 23 - 25 show the corona current-voltage characteristics of central and the two outer wires to the right of the central one of 5-wire precipitator as influenced by the wire radius at the same plate-to-plate spacing. Similarly, Figs. 26 - 28 show the corona current-voltage characteristics of central and the three outer wires to the right of the central one of 7-wire precipitator as influenced by the wire radius at the same plate-to-plate spacing. It is quite clear that the corona current-voltage characteristics is the lowest for the central wire and moves up for outer wires in the direction toward the end of the collecting plates. This is because the central wire is fully shielded by the other wires with a subsequent decrease of the field at its surface and increase of onset voltage [5, 6]. At the central wire, the
Fig. 7. Error currents versus applied voltage.

Fig. 8. Effect of plate-to-plate spacing on current-voltage characteristics of one-wire ESP ($r_c = 0.26$ mm).

Fig. 9. Effect of plate-to-plate spacing on current-voltage characteristics of one-wire ESP ($r_c = 0.935$ mm).

Fig. 10. Effect of plate-to-plate spacing on current-voltage characteristics of one-wire ESP ($r_c = 1.975$ mm).

Fig. 11. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of three-wire ESP ($r_c = 0.26$, $d = 14.5$ cm).

Fig. 12. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of three-wire ESP ($r_c = 0.935$, $d = 14.5$ cm).

Fig. 13. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of three-wire ESP ($r_c = 1.975$, $d = 14.5$ cm).

Fig. 14. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of five-wire ESP ($r_c = 0.26$, $d = 14.5$ cm).
Fig. 15. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of five-wire ESP (r_c = 0.935 mm, d = 14.5 cm).

Fig. 16. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of five-wire ESP (r_c = 1.975 mm, d = 14.5 cm).

Fig. 17. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of seven-wire ESP (r_c = 0.26 mm, d = 14.5 cm).

Fig. 18. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of seven-wire ESP (r_c = 0.935 mm, d = 14.5 cm).

Fig. 19. Effect of plate-to-plate spacing on current-voltage characteristics of central wire of seven-wire ESP (r_c = 1.975 mm, d = 14.5 cm).

Fig. 20. Current-voltage characteristics of central- and outer- wires of three-wire ESP (r_c = 0.26 mm, d = 14.5 cm, 2H = 40 cm).

Fig. 21. Current-voltage characteristics of central- and outer- wires of three-wire ESP (r_c = 0.935 mm, d = 14.5 cm, 2H = 40 cm).

Fig. 22. Current-voltage characteristics of central- and outer- wires of three-wire ESP (r_c = 1.975 mm, d = 14.5 cm, 2H = 40 cm).
onset voltage increases with the increase of the number of discharge wires, as the shielding effect becomes more pronounced with the increase of the number of discharge wires, Fig. 20-28. The shielding effect decreases in the direction toward the end of the collecting plates. This is why the current per discharge wire in a multi-wire precipitator increases toward the end of the collecting plates for the same applied voltage and same radius of the discharge wires, Fig. 20-28.

VIII. CONCLUSIONS

(1) Corona current-voltage characteristics of a laboratory electrostatic precipitator with 1, 3, 5 and 7 discharge wires are calculated and measured for each wire in the laboratory. The calculated values agreed reasonably with those measured experimentally.

(2) The smaller the wire radius, the smaller the onset voltage and the higher is the corona current at the same applied voltage and plate-to-plate spacing for single- and multi-discharge wire precipitators.

(3) The larger the plate-to-plate spacing, the higher the onset voltage and the smaller is the corona current at the same applied voltage and wire radius for single- and multi-discharge wire precipitators.

(4) The larger the number of discharge wires, the more is the shielding effect imposed on the central wire.
with a subsequent increase of the corona onset voltage and decrease of corona current emitted from the central wire at the same applied voltage and plate-to-plate spacing.

(5) The current per discharge wire in a multi-discharge wire precipitator increases toward the end of the collecting plates for the same applied voltage and same radius of the discharge wires.

ACKNOWLEDGMENT

The authors would like to thank Prof. Mazen Abdel-Salam of Assiut University, Egypt for his interest in this research work.

REFERENCES


APPENDIX

The procedure for determining the value of \( \rho_e \) at the surface of the discharge wire where a flux line emanates is explained as follows:

(1) The mean charge density \( \rho_m \) along the flux line is computed along the flux line using the following equation:

\[
\rho_m = e_0 \int (V - V_0) \int \int \frac{d\eta}{\sqrt{\xi^2 + \zeta^2}} d\phi
\]  

(9)

(2) Two values of \( \rho_{e1} \) and \( \rho_{e2} \) are arbitrarily chosen such that \( \rho_{e2} > \rho_{e1} > \rho_m \).

(3) For each value of \( \rho_e \), the charge density distribution along the flux line (using eqn. (6)) and the corresponding mean charge density \( \rho_{m1}, \rho_{m2} \) are evaluated.

(4) Subsequent values \( \rho_{e1} \) are calculated using the following iteration function:

\[
\rho_{ei} = \rho_{e(i-1)} + \frac{\rho_m - \rho_{m(i-1)}}{\rho_{e(i-1)} - \rho_{e(i-2)}} (\rho_{e(i-1)} - \rho_{e(i-2)}) \quad i = 3, 4, 5, \ldots
\]

(10)

The corresponding value of \( \rho_{m1} \) is then calculated before proceeding to the next iteration. The procedure is continued until \( \rho_{m1} \) approaches \( \rho_m \) to the required precision \( \epsilon \).